YOUNG TYPE INEQUALITIES OF MEASURABLE OPERATORS

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Let $(\mathcal{M}, \tau)$ be a semi-finite von Neumann algebra, $L_0(\mathcal{M})$ be the set of all τ -measurable operators and $\mu_t(x)$ be the generalized singular number of $x \in L_0(\mathcal{M})$. We proved that if $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$, $r \geq 2 \min\{\frac{1}{p}, \frac{1}{q}\}$ and $x, y \in L_0(\mathcal{M})$, then the Young type inequality $\mu_t(|xy^*|^r) \leq \mu_t(\frac{1}{p}|x|^{pr} + \frac{1}{q}|y|^{qr})$, for all $t > 0$ holds.

Keywords: generalized singular number; τ -measurable operator; semifinite von Neumann algebra.

The classical Young inequality, in general, this inequality for operators does not hold. If $\mathcal{A}$ is a unital commutative C^* -algebra, then

$$|ab| \leq \frac{1}{p}|a|^p + \frac{1}{q}|b|^q, \quad \forall a, b \in \mathcal{A}$$

(see [1]). Ando obtained a generalization of the above inequality as the following: If $x, y \in \mathbb{M}_n$, then

$$s_j(xy^*) \leq s_j(\frac{1}{p}|x|^p + \frac{1}{q}|y|^q), \quad j = 1, 2, \dots, n.$$

In [4], Farenick and Manjegani extended the Ando's result as the following: if $x, y \in \mathcal{M}$, then

$$\mu_t(xy^*) \leq \mu_t(\frac{1}{p}|x|^p + \frac{1}{q}|y|^q), \quad t > 0,$$

where $\mathcal{M}$ is a semi-finite von Neumann algebra with a faithful normal semi-finite trace τ and $\mu_t(z)$ is the generalized singular number of $z \in \mathcal{M}$.

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