MIXED CAUCHY BOUNDARY VALUE PROBLEM FOR NONCHARACTERISTIC DEGENERATE HYPERBOLIC EQUATIONS

@ N. Kakharman
n.kakharman@math.kz

УДК 517.518

DOI: 10.33184/mnkuomsh1t-2021-10-06.73.

В данной статье мы рассматриваем следующую смешанную задачу Коши

$$Lu = u_{tt} - k(t)\Delta_x u(x, t) = f(x, t), \quad (x, t) \in D = \Omega \times (0, T), \quad \Omega \subset R^n,$$

где $k(t) \geq 0$. Доказывается разрешимость этой задачи в пространствах Соболева $W_2^2(D)$ при условии $\frac{f}{k} \in W_2^1(D)$, далее эта задача исследуется для уравнения с младшими коэффициентами.

Ключевые слова: Смешанная краевая задача Коши, гиперболическое уравнение, потенциал Ньютона.

In this paper, we consider the following mixed Cauchy problem

$$Lu = u_{tt} - k(t)\Delta_x u(x, t) = f(x, t), \quad (x, t) \in D = \Omega \times (0, T), \quad \Omega \subset R^n,$$

where $k(t) \geq 0$. We prove the solvability of this problem in Sobolev spaces $W_2^2(D)$ under the condition $\frac{f}{k} \in W_2^1(D)$, further, this problem is studied for an equation with lower-order coefficients.

Keywords: Mixed Cauchy boundary value problem, hyperbolic equation, Newton potential.

A number of works has been devoted to the study of the mixed Cauchy problem for noncharacteristically degenerate second-order hyperbolic equations, beginning with the work of M.L. Krasnov [1]. Later these works were generalized for general degenerate higher order equations by D.T. Dzhuraev [2], V.N. Vragov [3] and A.I. Kozhanov [4].

In the study of the mixed Cauchy problem in a cylindrical domain, the lateral boundary conditions are usually local boundary conditions of the Dirichlet type or periodic boundary conditions.

In the work of T.Sh. Kalmenov and D. Suragan [5], the boundary condition for the Newton (volume) potential was found, which is a new integro-differential self-adjoint boundary condition for the Laplace equation.

In this paper, we study the mixed Cauchy problem for one class of noncharacteristic degenerate hyperbolic equations using this boundary condition.

Kakharman Nurbek, Institute of Mathematics and Mathematical Modeling, (Almaty, Kazakhstan)

Unlike other works devoted to this topic, where solutions of the mixed Cauchy problem with different lateral boundary conditions of the considered problems are obtained in weighted spaces, in this paper, all solutions of the considered mixed Cauchy problems are obtained in classical Sobolev spaces.

Литература

1. *M. L. Krasnov* Mixed boundary problems for degenerate linear hyperbolic differential equations second order // *Matematicheskii Sbornik*, **91**:1 (1959), 29-84.
2. *T. D. Dzhuraev* Boundary value problems for equations of mixed and mixed-composite types // Fan, Tashkent, 1979.
3. *V.N. Vragov* Boundary value problems for nonclassical equations of mathematical physics // Novosibirski, NGU, 1983. (in Russian)
4. *A.I. Kozhanov* Linear inverse problems for a class of degenerate equations of Sobolev type // *Vestnik Yuzhno-Ural'skogo Universiteta, Seriya Matematicheskoe Modelirovanie i Programirovanie*. **11** (2012), 33-42.
5. *T. S. Kal'menov* , *D. Suragan* D To spectral problems for the volume potential // *Doklady Mathematics*, **80**:2 (2009), 646-649.