ON THE DIFFERENT SIZE OF THE WHITE NOISE PERTURBATION IN THE PROBLEM ON THE INVISCID LIMIT OF THE STATIONARY MEASURE FOR ONE MODEL OF THE ATMOSPHERE

@ Yu.Yu. Klevtsova

yy_klevtsova@ngs.ru

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It was obtained the sufficient conditions on the right-hand side and the parameters of the Lorenz model for a baroclinic atmosphere with white noise perturbation for the existence a limiting point for any sequence of the stationary measures for this system when any sequence of the kinematic viscosity coefficients goes to zero. We consider white noise perturbation proportional to the kinematic viscosity in different degrees. A similar result was obtained for the equation of a barotropic atmosphere and the two-dimensional Navier-Stokes equation.

Keywords: Lorenz model for a baroclinic atmosphere, white noise perturbation, inviscid limit of stationary measures, the two-dimensional Navier-Stokes equation

We consider the system of equations for the quasi-solenoidal Lorenz model for a baroclinic atmosphere

$$\frac{\partial}{\partial t} A_1 u + \nu A_2 u + A_3 u + B(u) = g, \quad t > 0, \quad (1)$$

on the two-dimensional unit sphere S centered at the origin of the spherical polar coordinates (λ, φ) , $\lambda \in [0, 2\pi)$, $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\mu = \sin \varphi$. Here $\nu > 0$ is the kinematic viscosity, $u(t, x, \omega) = (u_1(t, x, \omega), u_2(t, x, \omega))^T$ is an unknown vector function and $g(t, x, \omega) = (g_1(t, x, \omega), g_2(t, x, \omega))^T$ is a given vector function, $x = (\lambda, \mu)$, $\omega \in \Omega$, (Ω, P, F) is a complete probability space,

$$A_1 = \begin{pmatrix} -\Delta & 0 \\ 0 & -\Delta + \gamma I \end{pmatrix}, \quad A_2 = \begin{pmatrix} \Delta^2 & 0 \\ 0 & \Delta^2 \end{pmatrix},$$

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Yulia Kletsova (Federal State Budgetary Institution “Siberian Regional Hydrometeorological Research Institute”, Siberian State University of Telecommunications and Information Sciences, Novosibirsk, Russia)

$$A_3 = \begin{pmatrix} -k_0\Delta & 2k_0\Delta \\ k_0\Delta & -(2k_0 + k_1 + \nu\gamma)\Delta + \rho I \end{pmatrix},$$

$$B(u) = (J(\Delta u_1 + 2\mu, u_1) + J(\Delta u_2, u_2), J(\Delta u_2 - \gamma u_2, u_1) + J(\Delta u_1 + 2\mu, u_2))^T.$$

Also, $\gamma, \rho, k_0, k_1 \geq 0$ are numerical parameters, I is the identity operator, $J(\psi, \theta) = \psi_\lambda \theta_\mu - \psi_\mu \theta_\lambda$ is the Jacobi operator and $\Delta\psi = ((1 - \mu^2)\psi_\mu)_\mu + (1 - \mu^2)^{-1}\psi_{\lambda\lambda}$ is the Laplace-Beltrami operator on the sphere S . A random vector function $g = f + \eta$ is taken as the right-hand side of (1); here $f(x) = (f_1(x), f_2(x))^T$ and $\eta(t, x, \omega) = (\eta_1(t, x, \omega), \eta_2(t, x, \omega))^T$ is a white noise in t . In [1] it was obtained for existence of a unique stationary measure of Markov semigroup defined by solutions of the Cauchy problem for (1) and for the exponential convergence of the distributions of solutions to the stationary measure as $t \rightarrow +\infty$ the sufficient conditions on the right-hand side of (1) and the parameters $\nu, \gamma, \rho, k_0, k_1$:

$$k_0 < F(\nu, \gamma, \rho, k_1), \quad (2)$$

where $F > 0$ is some real function of the arguments ν, γ, ρ, k_1 .

Let $\{E_i\}_{i=1}^\infty$ be an orthonormal basis for the space H^0 (for definitions of H^0 see the third paragraph of [1]). As a random vector function η , we consider

$$\eta^\omega(t, x) = \frac{\partial}{\partial t} \zeta^\omega(t, x), \quad \zeta^\omega(t, x) = \sum_{i=1}^\infty b_i \beta_i^\omega(t) E_i,$$

where $\{b_i\}_{i=1}^\infty \in l_2^+$, and $\{\beta_i^\omega(t)\}_{i=1}^\infty, t \geq 0$, is a sequence of independent real Brownian motions with respect to some right continuous filtration $\{F_t\}_{t \geq 0}$ such that all the P -nullsets of the σ -algebra F lie in F_0 (for definitions of H^0, l_2^+ see the third paragraph of [1]).

Let

$$\begin{aligned} k_0 &= \nu k'_0, & k_1 &= \nu k'_1, & \rho &= \nu \rho', & f &= \nu f', \\ b_i &= \nu^\alpha b'_i, & \alpha \in \mathbb{R}, & i = 1, 2, \dots, & \{b'_i\}_{i=1}^\infty &\neq 0, & \zeta &= \nu^\alpha \zeta', \end{aligned}$$

where k'_0, k', ρ', f' and $\{b'_i\}_{i=1}^\infty$ are independent of ν .

In the present work we consider the different values of α and give the answer when there exists a limiting point for any sequence of the stationary measures for the system (1) when any sequence of the kinematic viscosity coefficients goes to zero and when it is absent.

A similar result is obtained for the equation of a barotropic atmosphere and the two-dimensional Navier-Stokes equation. A comparative analysis with some of the available related results is given for the latter.

Литература

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