NLS EQUATION WITH COMPLEX POTENTIAL: MODES AND PSEUDO-MODES

@ G.L. Alfimov, A.O. Slobodyanyuk, D.A. Zezyulin
galfimov@yahoo.com

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Недавно Ю. Коминис и соавторы [Хаос, солитоны и фракталы, **bf 118**, 222-233 (2019)] предположили, что нелинейное уравнение Шрёдингера (NLS) с комплексным потенциалом вида $W(x) = W_1(x) + iCW_{1,x}(x)$, поддерживает непрерывные семейства локализованных нелинейных мод. Здесь $C \in \mathbf{R}$ и $W_1(x)$ - действительный и ограниченный дифференцируемая функция. Мы называем эти потенциалы *W-dW-потенциалами* и подробно изучаем нелинейные режимы для NLS с W-dW-потенциалом. Мы показываем, что (i) эти режимы существуют только для малые амплитуды потенциала W-dW и являются только приближенными (*псевдомоды*); (ii) несмотря на этот факт, они устойчивы и демонстрируют динамику, которая описывается коллективным координатным подходом.

Ключевые слова: нелинейное уравнение Шрёдингера, нелинейные режимы, диссипация, четно-временная симметрия.

Recently Y. Kominis and coauthors [Chaos, Solitons and Fractals, **118**, 222-233 (2019)] have suggested that Nonlinear Schrödinger Equation (NLS) with complex potential of the form $W(x) = W_1(x) + iCW_{1,x}(x)$, supports the continuous families of localized nonlinear modes. Here $C \in \mathbf{R}$ and $W_1(x)$ is a real-valued and bounded periodic, quasiperiodic, etc.) differentiable function. We call these potentials *W-dW potentials* and study in detail the nonlinear modes for NLS with W-dW potential. We show that (i) these modes exist only for small amplitudes of W-dW potential and are approximative only (*pseudo-modes*); (ii) in spite of this fact, they are robust and exhibit the dynamics that is described by the collective coordinate approach.

Keywords: nonlinear Schrödinger equation, nonlinear modes, dissipation, parity-time symmetry.

There are two cases when the nonlinear Schrödinger equation (NLSE) with an external complex potential $W(x)$,

$$i\Phi_t = -\Phi_{xx} + W(x)\Phi \pm 2|\Phi|^2\Phi, \quad (1)$$

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Georgy Alfimov, PhD, MIET University (Moscow, Russia), professor;
Alexander Slobodyanyuk, MIET University (Moscow, Russia), master student;
Dmitry Zezyulin, PhD, ITMO University, (St-Petersburg, Russia), postdoc;

supports continuous families of localized stationary modes of the form

$$\Phi(x, t) = \phi(x)e^{i\mu t}, \quad \mu \in \mathbf{R},$$

$$\lim_{x \rightarrow \infty} \phi(x) = \lim_{x \rightarrow -\infty} \phi(x) = 0.$$

They are (i) the $\mathcal{PT}$ -symmetric potentials and (ii) the Wadati potentials. Recently Y. Kominis and coauthors, [1], have suggested that the continuous families can be also found for complex potentials of the form

$$W(x) = W_1(x) + iCW_{1,x}(x),$$

where C is an arbitrary real and $W_1(x)$ is a real-valued and bounded differentiable function.

For brevity, we call these potentials *W-dW potentials*. The results of our study for NLS with W-dW potentials are as follows.

(A) If W-dW potential is small (of order $\varepsilon \ll 1$) our asymptotic procedure stops at the terms of the ε^2 order. So we conjecture that no continuous families of authentic nonlinear modes exist in this case, but *pseudo-modes* that satisfy the equation up to ε^2 -error can indeed be found.

(B) For a W-dW potential well of finite depth we support the hypothesis of the point (A) with numerical arguments.

(C) The simulation of dynamics of the pseudo-modes show that if the amplitude of W-dW potential is small, the pseudo-modes are robust and display persistent oscillations around the certain position predicted by the asymptotic expansion.

(D) We found the authentic stationary modes of (1) which do not form a continuous family, but exist as isolated points. Numerical simulations reveal dynamical instability of these stationary modes.

The details of our study can be found in [2].

Литература

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